Disclosure

Earlier versions of this paper were flatly rejected by a couple of leading journals. This one may be somewhat easier to understand, but you will still be reading it on your own responsibility.

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Is There a "Periodical Table of Cities"?

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Abstract: Comparing cities on an international scale is currently hampered by a lack of common definition of a city, and by a lack of widely accepted limits for the minimum and the maximum city size, both in terms of population and its area. It seems that some Internationa Table of Cities, modeled after Periodic Table of Elements, where cities are differentiated by their size class, might provide a useful framework for such a comparison. Design of such a table, based on "the rule of threes", borrowed from the Central Place Theory, will be discussed in this paper.

Key words: city size-class, net urban area, urban population, densities, Central Place Theory,

1) Introduction

Why do we need some International Table of Cities?

After six thousand years of urbanization there is still no agreement to where, or rather, at what size the city starts, and at what size it ends. Notions of small, medium size, and large cities were of some help while cities were generally much smaller. At this point, when the urbanization at the world level has passed the fifty percent mark, and when the population of large cities is numbered in millions, such notions are not all that helpfull. Besides, vast majority of city size comparisons at international level are made disregarding the huge difference in their areas.

The confusion in perception of city size is still greater due to *different concepts of cities.* For instance, a journalist, often quoting some official statistical data, may claim that population of Athens is 5 million, of Buenos Aires 12 million, of New York more than 20 million, and of Tokyo more than 30 million. These figures are correct for *the larger urban areas*, but not for the cities proper. The population of mentioned ***cities*** is several times smaller, and *within their* ***administrative boundaries*** they respectively have: Athens – less than 1 million, Buenos Aires - 3 million, New York City and Tokyo - 8 million each. But *areas* within their administrative boundaries differ considerably too. Athens took just 38 km² to accommodate these 800.000 inhabitants; 3 million inhabitants of Buenos Aires are settled within first 203 km²; the 8 millions living in five boroughs of New York City are settled on 800km²; while 8 million citizens of central Tokyo are accommodated on 600km².

Yet if we would try to compare these cities within some ***common denominator***, let's say within *the core 300 km²,* the differences would be much smaller. At such a core territory of 300 km², in Athens live 2,4 million, in Buenos Aires 3,8 million, in New York City 4,9 million, and in Tokyo 4,5 million inhabitants. In other words difference between Athens and Tokyo have shrunk from more than 8:1 to less than 2:1. But currently we don't have such a common metric (a *"city-meter")* to compare cities, particularly at the international level.

The two main concepts of ***urban hierarchies*** (the Central Place Theory and the Rank-Size Rule) are not of much help in this regard. Firstly, both are focused just on the local urban hierarchies, whether regional or national. Secondly, the Rank-Size Rule is taking into account only the relative place of a single city within such a hierarchy, while the Central Place Theory, although regarding *ranks* as ***size classes*,** assumes only one city size in each of them.

Therefore, it seems that some framework for comparing cities of different size, which would explicitly refer not only to *their population number*, but also to their *areas* (and *densities*) as well, is long overdue. In designing such a framework, which may be labeled as ***the International Table of Cities*** (**ITC**), and modeled after **the Periodical Table of Elements** (**PTE**), there are several issues we have to address.

First, we will have to tackle the issue of the minimum and the maximum city size. Then, we will have to decide *how many* ***size classes*** (***rows)*** we should look for in such a table. Next, we will have to decide *how many* ***columns*** (size sub-classes) should be in each of the rows. Perceiving ***city as a set of sub centers*** might be helpful, since the difference among cities within the same size class (rows) will be reflected in the number of city sub-centers. Finally, the figures we get in the end for ***the city population*** will have to be checked to see how they match to the ones for ***the city areas***. In the process we may also get ***the density figures.***

The rest of paper is organized as follows: In the first section we will see what we can learn from the two main concepts of urban hierarchy (the Rank-Size Rule and the Central Place Theory). Design of the table, referring only to ***the size of the population*** will be discussed in the second section. In the third section, ***the area limits*** will be considered. The density issues will be addressed in the final section.

1) What can we learn from two main concepts of urban hierarchy?

a) The Rank-Size rule and the Zipf's Law

The Rank-Size Rule, a kind of mysterious Pareto law for Cities stresses a reverse correlation between city's rank in the urban hierarchy and its size in terms of population. The special case when the rank of a city tells us how much smaller its population is compared to the top ranked city is known as Zipf's Law (1949).

The problem with the Rank-Size Rule and the Zipf's Law was that they were not confirmed in reality in far too many cases, as was shown already by Hoyt (1951). Later it was confirmed by Stewart (1958) who found that Zipf's Law holds mainly for big countries with rather small amount of foreign trade /see also Rosen, Resnick (1980), and Nitsch (2005)/. Basically, there are two big problems. First, in many cases the top ranked city is simply too large (*primate city*), and, second, there is some critical city size in the lower tail of the urban hierarchy below which neither Pareto, log normal, nor Zipf don't hold particularly well.

Explanations for these "aberrations" from Zipf's Law were found in *too small size of the country* [Berry (1961)], which as Wheaton, Shishido (1981) found is in reverse proportion to the primate city dominance; in its *lower urbanization level* (Berry 1964), and so forth.

Much later Gabaix (1999) found that Zipf's Law holds quite well if *Gibrat Law* holds as well (that is, if the urban growth rate is not depended on the city size). But essentially Zipf's Law appears to have a problem with *too big* and *too small* cities. From Eeckhout (2004) diagrams for the upper and the lower tail of urban hierarchy, it seems that the problem arise when cities have a population of *more than 2 million*, as well as when they have *less than 10.000*.

Yet, even if all these problems were solved, and if Zipf's Law would perfectly describe all urban hierarchies, it still would say nothing about ***the spatial distribution of cities*** within the hierarchy.

b) Central Place Theory and spatial distribution of cities

Enters Walter Christaler (1933). He was aware of the Rank-Size Rule, but considered it to be a mere statistical gimmick. His goal was to formulate a ***general location theory*** (after von Thünen's, and Weber's respective partial contributions in the theory of location in agriculture, and industry), and, as we know, he did it with just three key elements.

First, he introduced central products and services provided in *central places* (to differentiate them from agricultural products – produced in the field, and not at one point); second, the market for these central products and services was determined by their *threshold* (minimal number of customers required to make the endeavour profitable), and their *range* (maximum transportation cost – in time and money – a customer was willing to suffer in order to buy the product); and finally, all central places were part of *an urban hierarchy*, where their (collective) *rank* (or, rather, a size-class) was determined by the bundle of central goods and services they were providing.

Visually, the Central Place Theory is still most remembered for the *hexagonal, lattice shape* of the central place markets in all of the ranks (size-classes), but more important is ***"the rule of threes"*** (Beckmann 1958). This rule means that, looking from the top, in each rank down the hierarchy there are 3 times more central places than in the preceding one. Or, in other words, that 3 central places of one rank are *inducing the emergence* of one central place of the next, higher rank. But, what often pass unnoticed, this also means that each central place of the next rank have *a **three times larger population*** than the central place of the current rank.

The assumptions for this very simplified theory are that all central places are located on a homogenous, endless plain, with customers having the same taste and the same purchasing power. Therefore, the network of cities would be **evenly spread** across the plain, each part of it having an equal share of the central places of all ranks. From the key assumptions follows that the sectoral mix would be the same for all Central places of the same rank (size-class).

Christaller (1933) himself made it clear that this *oversimplified picture* would not reflect reality in the more wrinkled, mountainous area. In such an area the number of central places in any rank would be smaller than in the endless plain. But this *"shortage of central places"* would be equally distributed throughout all the ranks. It goes without saying that the same would happen in a desert, or a coastal area.

For the first formulation of a general location theory, this was quite sufficient, but soon its simplicity started to be considered more as a liability than as an asset. Aside from positive comments of this explanation of spatial distribution of cities (central places), almost every researcher was trying to make some improvement, by relaxing some of the assumptions.

For example, Lösch (1954) have tried to give economic explanation of the Central Place Theory, suggesting that although all the shops were in monopolistic position, they were not earning monopolistic profits if the size of the market area was barely exceeding the threshold. He also noticed that the *assumption of sectoral mix being the same for all central places* within *the same rank* (size-class) is not realistic, at least for the smaller cities, which often have a different sector dominating the structure of the city economy.

Berry and Garrison (1958) confirmed that in each of the higher ranks, the bundle of central goods is much larger than in the previous one. Eaton, Lipsey (1982) in their economic interpretation of central places found that *multipurpose shopping* is leading to formation of *shopping centers*.

But many researchers found that the key element, "the rule of threes" doesn't correspond with reality almost anywhere. Therefore, much of the effort was invested in finding a more adequate ***"nesting factor"*** (telling us how many central places of lower size class are inducing the emergence of one central place of higher size class). For instance, Tinbergen (1968) formulated his version of Central Place Theory using nesting factor of 4 (which Christaller considered to be valid for the transportation).

Parr (1978), who looked at Central Place Theory from each and every angle, tried to relax the assumption of ***a fixed*** nesting factor, and found that values for it were ranging between 2 and 4. Dacey (1966) believed that *the nesting factor* was lower than 3, and constantly declining. Later Parr (1980) found out that Central Place Theory nesting factor of 3 don't hold anymore even in the Southern Germany.

After Woldenberg (1968), and Parr (1969), both following Lösch (1938), noticed that in reality "the Rule of threes" doesn't necessarily hold, a number of suggestions were made which abandoned the ***fixed*** nesting factor altogether. Even Beckmann and Christaller were ready to accept that the nesting factor should not have to be a fixed one. But not much was gained by accepting the ***shifting*** instead of a ***fixed*** nesting factor, since there was no one-fit-all sequence for a ***variable*** nesting factor.

It is more plausible that deviations from the fixed nesting factor ("The Rule of threes"), and a lack of central places in some of the higher ranks, should be considered as a temporary feature during the process of urbanization, until the complete 1-3-9 etc. sequence doesn't emerge.[1]

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[1] For instance if the top 3 ranks have a following distribution of cities: 1 in the first, 2 in the second and 10 in the third rank (size-class), the sequence would be 1-2-10, and it would look as if the urban hierarchy is way out of "the rule of threes". But, if one of those 10 cities in the third rank, in a foreseeable future, grows enough to reach the next, higher rank, this "irregular" pattern would turn into one perfectly following "the rule of threes", in an orderly sequence of 1-3-9.

Facing this obvious lack of central places in almost every size-class, Loesch (1954) even tried to save "the rule of threes" by assuming that each central place of higher order performs the role of central places of all lower orders. Thus instead of Christaler's distribution (1-3-9-27-81-243 etc.) he got a more modest distribution (1-2-6-18-54-162 etc.) with proportionally lower number of central places in each size-class. This Loeschian distribution was not much better in describing the real world distribution of cities, but it is worth noting that it may be regarded as a kind of not fully evolved Christalerian distribution of central places.

The price for all the suggested improvements was ***a loss of simplicity*** which is the key asset of the Central Place Theory (pretty much like the assumptions in the Tiebout's concept of the Local Public Good). In the end Parr (1980) concluded that, yes, it is rigid, but this is because it is more an ideal, *end picture*, than the description of the current situation at any point of time – until the process of urbanization is over. Then a stable spatial equilibrium may emerge, and it is this end picture that Central Place Theory is portraying. Therefore it should be regarded as more successful in describing the spatial distribution of cities in countries where urbanization process is closer to the end, than to the beginning.

For the design of a Periodic Table of Cities it is important that ***the "rule of threes"*** gives us a fairly close estimate of the number of cities in each rank (size-class), and gives us an idea of how many ranks there may be in each urban hierarchy. No other fixed, and no shifting, nesting factor proved to be closer to reality. Yet it is obvious that the "rule of threes" is not too successful in "predicting" the actual number of cities in each of the ranks *exactly* – but only *roughly*, pretty much like the Zipf's Law.

c) Drawers in the Christaler's Equal Hierarchical Population (E.H.P.) Table

Despite all complaints about the Central Place Theory's unrealistic assumption about all cities within the same rank (size-class) being of the same size (in terms of population), almost no one, save for Beguin (1982), bothered to state the obvious – that in such a case *the total population number of all cities in each rank* – is equal to the population of the city in the top rank.

Actually Parr (1970) had even given a name to such a class of urban hierarchy –**Equal Hierarchical Population (E.H.P.)** model, but he only briefly described it, without any further comment. Somewhat later, Richardson (1973) merely mentioned this (E.H.P.) possibility as a special case in a similar review of various distribution sizes.

Yet in this E.H.P. case we may compare such an urban hierarchy to a desk (table) with drawers of *exactly the same size*. ***The total population number*** in each city size class is equal to the population of the most populous city in the hierarchy. Therefore the E.H.P. feature of urban hierarchy in Christaller's Central Place Theory is pretty much a kind of ***Zipf's Law for the city size-classes***. A kind of a "team event".

Christaler's E.H.P. Table may be useful in the analysis of and ordinary, centralized urban hierarchy. Namely, if drawers of the table are not of the equal size, this may indicate a problem. For instance, if some drawer (city size-class) is much narrower than others it indicates uneven development within urban hierarchy. This usually means that cities of some lower order have not yet induced the emergence of a sufficient number of cities of the next higher order. More frequently, the problem would be found in disproportionate size of the top drawer, indicating that the top ranked city is a "primate" city. On the contrary, in the case of decentralized urban hierarchies, such primate, or at least a distinguished single top city is lacking altogether.

For this paper it is important to notice that the number of drawers in this Christaler's E.H.P. Table may provide us with a starting point in search for the Periodic Table of Cities, by giving us an idea how many number of rows we should look for in such a table.

d) Christaler-Zipf's A.E.H.P. Table

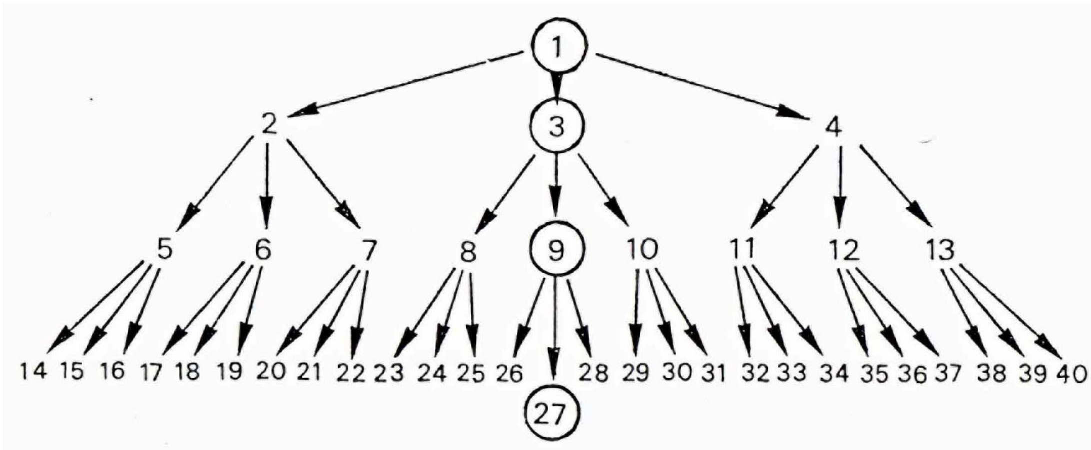
Whatever the usefulness of Christaler's Drawer Table it is still based on an unrealistic assumption of equal size of all cities within the same size-class. Since Christaler's E.H.P. Drawer Table can be regarded as some kind of a Zipf's Law for the city ranks (size-classes) in urban hierarchy, it may be worth a wile to check the connection between Christaler and Zipf, which was noticed rather early.

Already Lösch (1938) saw a similarity of Central Place Theory to Rank-Size Rule. Then Hoover (1955) noticed similarity with Zipf's Law. But it was Beckman (1958), who was first to suggest that cities within each rank don't have to be of equal size, and hinted that they may differ so much as to actually come close to the Zipf's distribution. At the same time Berry and Garrison (1958) made explicit comparison of the two concepts and found that they are compatible.

It took a while until Derycke (1979), have shown graphically the connection between Christaler and Zipf, (as can be seen in picture 1.0.). A few years later Beguin (1985), in order to illustrate the Beckman's (1958) suggestion about the similarity between Zipf's and Christaller's size distribution, calculated how much the cities within the same rank (size-class) should differ in size from the average one.

Thus, when we translate Zipf's distribution into Central Place Theory's "rule of threes" we get somewhat modified Christaller's E.H.P. Table, where cities within the same size-class, don't have to be of equal size. ***The Christaller-Zipf's Drawer Table*** thus brings a more realistic assumption about the size of each city. The price tag for this gain is that all the drawers would be slightly larger than the top one (up to 10%), and would slightly differ among themselves.

Picture 1.0. Graphical connection between Central Place Theory and Zipf's Law



Source: Derycke (1979)

The consequence for the analysis of urban hierarchy would be that not all drawers (combined population of cities within each size-class) would have to be of ***exactly*** the same, but only of ***roughly the same*** size in this team event, thus turning the E.H.P., into an ***almost*** E.H.P. (A.E.H.P) table. This in turn means that only a ***considerably*** larger (or smaller) "drawer", than the one for the top ranked city, would indicate some problem within that particular size-class of the urban hierarchy.

Yet, despite this possible usefulness in analysis of centralized urban hierarchies, Christaller- Zipf's A.E.H.P. Table would still be fit only for comparison within the urban hierarchy of *a single country*, and not for comparison of all cities on an international scale. Therefore we still need some ***ITC***, which may borrow some features from Christaller- Zipf's Drawer Table, particularly the suggestion about the maximum number of rows.

But before we get to this question of the number of rows, we have to tackle the issue of the minimum and the maximum city size. Then, along with the number of rows, the number of columns has to be decided, as well as the size of the *corresponding areas* for different city population numbers. Finally the issue of big variation in city densities have to be tackled too. In the next section we shall start with the population figures.

2) City Population numbers (City Limits)

The literature on city limits in terms of population numbers is almost non-existent, so instead of references from some literature overview, prepare to see a lot of guesses and arbitrary choices.

2.1. ***The minimum city size***

Unfortunately, there is ***no universally accepted definition of a city***.[2] The minimal size of a city, in terms of population, which divides it from rural settlements, is a rather tricky, even diplomatic issue. Less densely populated countries tend to lower the threshold which divides urban from rural places, in order to present them as more urbanized. Therefore they recognize cities in settlements with as little as 2.000 (or even fewer) inhabitants. On the other hand, more densely populated countries raise the bar much higher, ending up with large villages, of about 10.000 inhabitants.

It is deplorable that urban theory has not provided some ***concept of the minimal bundle of essential goods and services***, required to be delivered on a daily basis, as a precondition for some settlement to be labeled as urban. Defining such a bundle (let's say access to basic utilities such as running water, paved roads, or electricity) would not prevent further diplomatic disputes, but would certainly help as a guideline. Discussion between urban theoreticians and politicians would possibly result in a compromise, which would hopefully make the rules for deciding a minimum city size more transparent and acceptable to all.

In absence of such a theoretical concept, this diplomatic dispute over the minimum city size in terms of population can only be addressed diplomatically. A solution suggested here is to consider all settlements with population of *more* than 10,000 as cities; and to label as ***"micro-cities"*** all settlements with population of *less* than 10,000 which are by their countries, under their own criteria, proclaimed as urban.[3]

2.2. ***The maximum city size***.

Contrary to the issue of the minimum size, the question of the maximum city size has rarely been discussed at all. So, there is a vast array of nicknames for larger than urban territories (Urban Areas, Urban Agglomerations, Urbanized Areas, Metropolitan Areas, Larger Urban Zones, Urban Regions etc.), without any indication (except for Gottmann's Metropolis), where there may be a cut-off line between the city and some supra-urban territory. Or, rather, there are only pragmatic decisions of local authorities, which occasionally move the city boundaries outwards.

Generally, such border line divides the periphery of the city from its suburbs. That border line is quite obvious, particularly in the case where the suburbs are mainly dormitories, lacking even some essential urban services like parks, corner shops and

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[2] Overview of different definitions of cities can be found, for example, in UN (1987), "The prospects of World Urbanization", UN, New York, Population Studies No. 101.

[3] Why is population of 10,000 taken as a threshold between the "micro" and the "regular" cities?
(1) Higher figure for the maximum is rarely mentioned even in very densely populated countries
(2) Villages with population of more than 10,000 are extremely rare, and may not last for long – as villages. Either their population will migrate elsewhere, or a couple of factories and elementary utility networks will transform them into proper cities.

meeting points. On one side there are urban zones of the ***Inner City area*** which we may label as *the Inner Center*, *the Wider Center* and *the Periphery*. On the other, there are *the Inner* and *the Outer* ***Suburban Belts***, spread on a far larger territory than the Inner City, quite often by far exceeding the 10:1 ratio.

Administrative divisions within the wider urban areas often make a distinction between the city proper and its suburbs. Still, there are no universally accepted guidelines regarding both *the maximum population number*, and *the maximum area for such Inner City (City Proper, Core City)*. As a consequence, there are huge differences in official size of the Inner Cities, from less than 100 km² to over 1,000 km².

On the other hand, there is a lot of data on the ***metropolitan areas*** (mainly based on the concept of the urban job market). But they are of no real help in this regard, since they are not based on fixed limits (either spatial, or in terms of population), for all metro areas. By watering down the criteria for delineating *the urban job market*, metro areas started to grow beyond any proportion to the Inner City area.

Therefore, for the time being, we will have to rely on ***some rule of thumb*** and arbitrarily define the maximum city size. To reach such a maximum *in terms of population*, we will first have to define such a maximum *in terms of the city area*. In order to narrow the room for error in such an arbitrary decision we will have to commensurate the three key zones of the Inner City: *the Inner Center*, *the Wider Center* and *the Periphery*. For most of the biggest cities their Inner Center and Wider Center area combined don't exceed 100 km² (with a radius of less than 6 km, which is about an hour walk from the focal point in the city). The question then is how much larger the periphery can be, compared to the Inner and the Wider Center zones combined?

It seems plausible that the Peripheral Zone of the Inner City can be equal to, or double the size of the combined area of the Inner, and the Wider City Center. This suggests that the limits for the Inner City area should be looked for between 100 km² and 300 km² marks. For instance in Paris, even the peripheral zone lies within the first 100 km². But Paris has a rather high density of two hundred persons per hectare. At the other end the territory of the Inner London is somewhat larger than 300 km², with density less than half of the one in Paris.

If we then consider how many people live in these first 300 km², we will find that within this area only the fifty most populous cities are inhabited by more than three million people. So, we may roughly take this three million figure as a proxy for a maximum population number for a city, considering those top fifty cities as exceptions, or a special case – "***extra large cities***", pretty much like the one regarding the "micro cities".

We may also notice that lower and upper population number limits are pretty close to the ones we can read from those Eeckhout (2004) diagrams, but we shouldn't read too much into this similarity. Or, should we?

2.3. The number of rows (size-classes)

By defining the lower and the upper city limit in terms of population, and accepting "the Rule of threes", we have practically decided *the number* of size-classes, that is, the rows in our ITC! Namely, if we take the city with a population of 10.000 as a threshold size for the "micro-city" rank, and 3 million as a threshold for the top city size-class, there may be only 6 size classes (rows) in the table, or 7, if we add one for the "extra large" cities.

When we borrow a multiplier of 3 from Central Place Theory and start from the threshold of 10,000 inhabitants for the smallest city size class, it would set the threshold for the next size class at – 30,000, the threshold for the third size-class at – 90,000, for the fourth at – 270,000, for the fifth at – 810,000, for the sixth at – 2,430,000, and for a possible seventh size-class at – 7,290,000 inhabitants. So, six or seven size-classes (rows), with possibly one more, for cities bigger than 7.3 million inhabitants seem to be more appropriate for such a table.[4]

Since these threshold figures do not look particularly impressive or memorable we may round them up, by using intermittently the multiplier of 3, and the multiplier of 3,3, to reach the 10,000 – 30,000 – 100,000 – 300,000 – 1,000,000 – 3,000,000 sequence, with eventually another threshold of 10,000,000 inhabitants.

2.4. The number of columns ("subclass sizes")

If the Central Place Theory helped us with "the rule of threes" in deciding *the number of rows* (city size-classes) it is of no help regarding *the number of columns* (city size subclasses). Namely, "the rule of threes" in the Central Place Theory means that the number of cities in each rank (size class) ***progressively*** increases (1 – 3 – 9 – 27 – 81 – 243 etc.), which is normal and logical for urban hierarchy, but for some ITC we need a fixed, ***single digit*** number of city size sub-classes in ***each*** row.

If we would try to emulate the PTE, and have a ***constant increase*** in population numbers between cells throughout the PTC it would be too big for the lower ranks, and too small for the upper ranks.[5]
Therefore, we have to adjust the population increase between cells for the each size class. In fact it would be best to regard these population increases as ***additional sub-centers in***

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[4] This is not something particularly new. For instance Berry (1961) also uses 6 size-classes, albeit somewhat different ones (20,000-50,000, 50,000-100,000, 100,000-250,000, 250,000-500,000, 500,000-1 million, over 1 million), and only in context of national urban hierarchies.

[5] Let's start with the constant increase for the entire table. If we take the maximum city size to be ten million, and the minimum size to be 2,000 inhabitants, then, in order to have less than 100 cells in the table, the size of additional city sub-center would be equal to 9,998.000 : 100 = 99,800 or roughly 100,000 inhabitants! This is obviously far too much for the lower ranks (size-classes).
But, even if we reduce the maximum city size to one million, and increase the minimum city size to 10,000 inhabitants, the size of additional city sub-center would be reduced to 9,900 (10,000) inhabitants. This figure would still be too large for the lower ranks (size classes) and quite irrelevant for the top ones.

intra-urban hierarchy. And, since we have applied “the rule of threes”, the size of these additional sub centers will be interminently 3 or 3,3 times bigger in each of the higher size-classes. For the smallest size-class, additional city sub-center have a population of 1.000, for the next one 3.000, the third one – 10.000, and so forth in a sequence 1.000 – 3.000 – 10.000 – 30.000 – 100.000 – 300.000, and for the supra-urban levels 1.000.000 and 3.000.000. Of course it is rather easy to notice that these sub-centers are just somewhat disguised central places, transferred into the intra-city framework.

It is tempting to differentiate columns in such a manner to accommodate besides the plain cities, also for different groups of cities like mountain (or high altitude cities), island cities, coastal cities, desert cities, and provide each of them with a separate column, as is the case with the inert gasses or halogen elements in the PTE. But although there are obvious common features among them it is hard to establish a firm connection between each of these groups of cities and the number of city sub-centers lacking (in comparison to cities located in plains). So, we will have to consider city sizes in the table irrespective of geographical specifics of their location.

2.5. Designing issues

In attempt to find an ideal candidate for the ITC a logical choice may be the table 0.1. where all figures are well rounded, with thresholds at 10.000, 100.000, 1 million and 10 million inhabitants. The first row is obviously covering the “micro” cities, the next one may be regarded as the one with cities of small size, cities in the third row may be labeled as of medium size, and the ones in the final row as big cities. And it seems that the case is closed.

Yet, the multiplier in this table is too high. It is 10 instead somewhere in the range between 2 and 4. In other words it would take the existence of 10 city sub-centers of one level to induce the appearance of 1 city sub-center of the next higher level.

Table 0.1. Small number of rows - harsh transition

population (000)								
2	3	4	5	6	7	8	9	10
20	30	40	50	60	70	80	90	100
200	300	400	500	600	700	800	900	1,000
2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000

This doesn’t seem to correspond to reality. Usually, we may expect some deficit in sub centers of lower rank, so that there might be found one or two instead of three. But if five or more sub centers are required to induce the emergence of just one of the next, higher rank, this would imply that such a city is rather incoherent or even disintegrated.

This problem is reflected also in harsh transition in size of additional sub-centers (for instance from 1.000 in the first row, all of a sudden it is 10,000 in the next row), so that the first city in the next row (for instance that of 20,000) is twice the size of the threshold, (the maximum city size in the previous row). In other words if *the additional sub center* is the size of *the entire set of sub centers of the preceding city*, the transition from one row (rank, size class) to the next is too harsh. To soften it we should look for much smaller multiplier.

The Central Place Theory suggests the range for the multiplier to be between 2 and 4. Yet, if, following Davis (1970), we take the multiplier of 2, we would end up with too many rows and columns. The sequence of threshold populations would be: 10,000 - 20,000 - 40,000 - 80,000 - 160,000 - 320,000 - 640,000 - 1.280,000 - 2.560,000 - 5.120,000 - 10.240,000 resulting in 11 size classes (rows).

If we, following Tinbergen (1969), use the multiplier of 4, the number of rows and columns would be acceptable, but the problem may emerge in the next step, when we add areas to the rows, so that cities in each of the columns would not have the same density.

Therefore it appears that the mentioned sequence 10,000 – 30,000 – 100,000 – 300,000 – 1.000,000 – 3.000,000 with the multiplier interminently shifting from 3 to 3,3 matches best the purpose of the ITC. It seems that if you start applying the “rule of threes” you have to stick to it to the final picture.

It is worth noting though that, in a way, we may consider this sequence as an improved version of previous Table 0.1. That is, a version in which we have cut each size class into two, providing softer transition to the next size class. Thus both odd and even rows, taken separately, have a multiplier of 10, and only combined they have this multiplier interminently shifting from 3 to 3,3. Additional sub centers in the transition to the new size-class (row) are now only one third (or one fifth!) of the size of the previous threshold city size.

Regarding traditional city size categories, (small, medium, big), after one row for the “**micro**” cities, the small and the medium size cities are to be found in two rows each, one row is left for the big cities, and one is left for the “**extra-large**” cities above 3 million mark. Therefore ITC for population numbers may look as in the Table 1.0.

Table 1.0. Population numbers

Rank	population (000)						
	1	2	3	4	5	6	7
I	4	5	6	7	8	9	10
II	12	15	18	21	24	27	30
III	40	50	60	70	80	90	100
IV	120	150	180	210	240	270	300
V	400	500	600	700	800	900	1,000
VI	1,200	1,500	1,800	2,100	2,400	2,700	3,000
VII	4,000	5,000	6,000	7,000	8,000	9,000	10,000
VIII	12,000	15,000	18,000	21,000	24,000	27,000	30,000

What does these figures mean, and how should we read this table?

First, the blue figures denote ***city population*** and the white ones the ***population of supra-city areas***. These figures obviously don't cover all the possible city sizes, which means that a figure in a cell is covering *the entire interval between the cells*. For instance, a city of 200,000 inhabitants is represented by the cell for a city with population of 210.000. But this also means that, looking from the previous city size (sub-class) of 180.000, the additional intracity sub-center of 30.000 is *not yet fully developed*, but only up to 2/3.

From the Table 1.1. we can see that *at the threshold population* the size of the market becomes so big that it leads to the creation of *a city sub-center of the next higher rank*. Thus the farthest column to the right, the one where the threshold is reached, actually does not belong to this, but to the next, higher "period" (size-class), where a new city zone is added.

It is worth noting that "smuggling" central places as sub centers within cities, helps us to perceive city as a set of sub centers. This is not a novelty since Harris and Ullman (1945) were probably the first to regard city as a set of sub-centers. Later, Lowry (1964) based his entire model of metropolis on this premise, while von Böventer (1976) was explicit in analysis of the entire metro area in terms of sub centers. A mathematical interpretation of a city as a set of sub centers was given by Fujita, Krugman, Mori (1999).
But as we already noticed, "smuggling" central places as sub centers within cities, in a way, transfers the Christalerian urban hierarchy within the city borders. If then a distribution of these city sub centers within urban area can be predicted fairly well, it would be a sign of regularity expected from a periodic table.

Yet, as Eaton and Lipsey (1982) in their economic interpretation of Central Places stressed, multipurpose shopping is leading to formation of shopping centers. And shopping centers in particular and *the scale economy in retail trade* in general, have reduced considerably the number of smaller retail outlets, which are essential features of city sub centers at any level.

Parr (1995) looking for an *economic law of market areas*, connected Christaler with Reilly, which enabled him to explain better the variation of the size for central places of the same rank, and more importantly, to explain the uneven distribution of central places, where those of higher ranks are expanding at the account of those of lower ranks. Substituting city sub centers for central places we can, therefore, assume that the figures in the table 1.1. represent more *a possible maximum number of sub centers* within each rank (size-class), while the actual number have to be much lower.

Table 1.1. Number of sub-centers (population in 000)

3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
							1x10
1x3	1x3	1x3	2x3	2x3	2x3	3x3	3x3
10	**12**	**15**	**18**	**21**	**24**	**27**	**30**
							1x30
1x10	1x10	1x10	2x10	2x10	2x10	2x10	3x10
3x3	4x3	5x3	6x3	7x3	8x3	9x3	10x3
30	**40**	**50**	**60**	**70**	**80**	**90**	**100**
							1x100
1x30	1x30	1x30	2x30	2x30	2x30	3x30	3x30
3x10	4x10	5x10	6x10	7x10	8x10	9x10	10x10
10x3	13x3	16x3	20x3	23x3	26x3	30x3	33x3
100	**120**	**150**	**180**	**210**	**240**	**270**	**300**
							1x300
1x100	1x100	1x100	1x100	2x100	2x100	2x100	3x100
3x30	4x30	5x30	6x30	7x30	8x30	9x30	10x30
10x10	12x10	15x10	18x10	21x10	24x10	27x10	30x10
33x3	40x3	50x3	60x3	70x3	80x3	90x3	100x3
300	**400**	**500**	**600**	**700**	**800**	**900**	**1,000**
							1x1000
1x300	1x300	1x300	2x300	2x300	2x300	3x300	3x300
3x100	4x100	5x100	6x100	7x100	8x100	9x100	10x100
10x30	13x30	16x30	20x30	23x30	26x30	30x30	33x30
30x10	40x10	50x10	60x10	70x10	80x10	90x10	100x10
100x3	133x3	166x3	200x3	233x3	266x3	300x3	333x3
1,000	**1,200**	**1,500**	**1,800**	**2,100**	**2,400**	**2,700**	**3,000**
							1x3000
1x1000	1x1000	1x1000	1x1000	2x1000	2x1000	2x1000	3x1000
3x300	4x300	5x300	6x300	7x300	8x300	9x300	10x300
10x100	12x100	15x100	18x100	21x100	24x100	27x100	30x100
33x30	40x30	50x30	60x30	70x30	80x30	90x30	100x30
100x10	120x10	150x10	180x10	210x10	240x10	270x10	300x10
333x3	400x3	500x3	600x3	700x3	800x3	900x3	1000x3

3) City Area

City size has almost exclusively been dealt with in terms of population. The question of *city area limits* has been discussed much less. Or, rather, focus was on how far limits of a greater city area can be stretched, as in the concept of job market, to capture the entire urban hinterland.

In a way, *the literature on the size of market areas* was tackling this question. But again it is of not much help in defining the maximum city area. For instance Parr, after his analysis of market areas [Parr (1995)], made a logical step further and in later article [Parr (2007)] gave an overview of British experience with different spatial definitions of a city. Particularly for the greater city area. He concluded that overlapping of all these concepts would give us a relevant city area. Yet there is no hint at what the maximum of such area may be.

Rozenfeld et al (2011), in a similar attempt, started from some *minimal density* as a criterion for defining ***a continuously built area*** within the city and its surroundings. But they seem to be too flexible in defining city limits, since they were regarding *discontinuities* within the built area of 1 km or more as acceptable. In this way they were actually trying to smuggle the greater, ***gross urban area*** into a much widened concept of the net urban area.

Bertaud (2004) on the other hand, looks for the *continuously built area* in a much more narrow sense adding only *a maximum of 4 km² of adjacent green spaces*, to what we may consider as ***the net urban area***. He had much smaller areas in mind than Rozenberg et all (2011), since he usually compared cities at their first 100 km², 300 km², or 500km². In this way he actually suggested that we should look for ***the maximum city area***, or should we say ***the maximum of the inner city area***, within rather narrow range.

Lee and Holmes (2007), without any reference to Bertaud, have chosen **100 km²** (6 by 6 miles squares) *area limit* to test whether Zipf's Law hold for the areas of the same size. Although they were not suggesting that this 100 km² area is any kind of a maximum for the inner city area, this area size seems to be too narrow for comparison of the most populous cities in the world (particularly since they assumed gross city area). Therefore it looks as we have to choose between the 300km² and the 500km² as the threshold area.

If we turn to ***the actual administrative borders*** to help us in such a choice it would be to no avail. Namely, current administrative borders are widely varying for the cities of the same class size in terms of population. Or, rather, *their net urban areas* are pretty much of the same size, but it is *their hinterlands* that vary widely. Therefore if we focus on ***the net urban area***, covering the street grid (that is, with sidewalks, not roads), we are back to the choice between 300km² or 500km² area. Using the same approach as in the procedure to determine the maximum city size in terms of population (2.2.), and sticking to "the rule of threes", as we did in similar arbitrary decisions, it makes sense to use the 300km² area for such a yardstick.

(For an extensive list of cities at their first 300 km² see the Annex I)

When we have decided for the yardstick (a "city-meter") to compare first all of the most populous cities in the world, we can then define smaller areas to compare cities within all other size classes.

3.1. Which *areas* correspond to each of the size classes?

As we have accepted 300 km² as a maximum for the city area, and we have agreed that there are six size classes for inner cities in the table, it may be tempting to divide this 300 km² area by 6 and then to increase the maximum for each area by 50 km², to get a 50 km² - 100 km² - 150 km² - 200 km² - 250 km² - 300 km² sequence.

However that would be clearly wrong. Cities with small population need rather small areas, with far fewer urban functions to accommodate than the larger ones (for instance with no real need for large parks). From the Central Place Theory it follows that the number of city functions grows exponentially with the city size (in terms of population), demanding proportionally larger areas, as Woldenberg (1979) has noticed. On the other hand, the density, usually, also rises with the population number, with mixed effect. Namely, higher buildings reduce the need for additional space, but they bring higher traffic demand which dents considerably into such savings.

Therefore, we may tentatively assume that area requirements grow roughly in line (and not exponentially) with the rise in population number. Thus starting from the maximum area of 300 km² for the final row (a size class) in the table and going backwards, we can assume that the area maximums for the previous size-classes would respectively be 100 km² - 30 km² - 10 km² - 3 km² - 1 km².

In practice though, this is not so easy to detect, since ***the administrative territory of a city*** is usually confused with ***the area of the municipality*** (which is much larger, containing the entire hinterland). So, this correlation between the population number and the corresponding area is not so clearly visible from the official data, particularly for the smaller cities.

Things are not much better when we look at the urban units of larger size classes, where the picture is blurred by this silly competition for the largest possible hinterland which have stretched the concept of hinterland too much. Therefore, we have to separate the Inner City, from the Greater City Area, in order to see this correlation between the population number and the corresponding area more clearly. Adding this area limits to the population numbers from the table 1.0. we get the data presented in the table 2.0.

Table 2.0. Population numbers and area sizes								
Rank	Area	population (000)						
	km²							
I	1	4	5	6	7	8	9	10
II	3	12	15	18	21	24	27	30
III	10	40	50	60	70	80	90	100
IV	30	120	150	180	210	240	270	300
V	100	400	500	600	700	800	900	1,000
VI	300	1,200	1,500	1,800	2,100	2,400	2,700	3,000
VII	1,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000
VIII	3,000	12,000	15,000	18,000	21,000	24,000	27,000	30,000

Figures indicated as area limits for each size class, are denoting ***the range*** of areas between the previous and the current rank. So, the cities in the second size-class (with population between 12,000 and 30,000) have a net urban area between 1km² and 3km², while the cities of the top rank (with population between 1,2 and 3 million) have a net urban area between 100 km² and 300 km², and so on.

3.2. The area limits for the larger than city concepts?

By separating *the net urban area* from its hinterland, we have set the maximum to the (inner) city area, but this area is vitally interconnected to its hinterland, which varies very much in size. Currently many concepts are covering this *wider urban area*: greater city, urban district, larger urban zone, NUTS, metropolitan area, urban agglomeration, urban region etc., but none of them (except NUTSs) is specified in terms of a ***fixed*** area range. Any one of them may vary from several hundred, to over 10,000 km². In order to facilitate international comparison it seems recommendable to introduce some territorial ranges for these supra-city units too.

Once again the "Rule of threes" may do the job. Namely, if we accept this 1 – 3 – 10 – 30 – 100 – 300 km² sequence for the inner city, it is logical to add other thresholds at 1,000 - 3,000 - 10,000 - 30,000 km² for different levels of city hinterland. Beyond 30,000 km², by Gottman (1961) definition, we are facing ***megalopolises***, and between 10,000 km² and 30,000 km² we are obviously speaking of ***urban regions***.

For the rest it is more important that we accept these (or similar) numerical ranges, than how are we going to label them. For instance area beyond 300km² and up to 1,000km² can be labeled as ***the Greater City*** (while for some of those fifty odd most populous cities this area of up to 1,000 km², may coincide with their net urban area). Between 1,000 km² and 3,000 km², where usually *the outer suburban belt ends*, we may use the term the

Metropolitan Area. The area between 3,000 km² and 10,000 km², we may call ***Urban Agglomeration***.[6]

To compare those larger-than-city concepts, it is useful to take a look at the "split-times" of those extra-large cities in the "Urban Marathon", presented in the Annex II

3.3. The Inner City zones

When area limits are added to the city size classes we can illustrate ***the different zones*** *within* the city too. Even for the micro-cities we can interpret the next row as their *hinterland*. For the next size class, the small cities with population of up to 30,000 and their net urban area of 3 km², besides their *suburban belt* (in the next 7 km²), a distinction can be made between *the city center* (with population of up to 10,000 within the central 1km²), and *the periphery* (where up to 20,000 inhabitants are living on remaining 2km²).

From the third size-class onwards, besides the *periphery* and the *suburban belt* it is possible to distinguish between *the inner* and *the wider center* (the first having the same population and spreading on the territory of *the entire center* in the previous size class, while the second is equal to the number of people and the area of *the periphery* of the previous size class).

Table 2.1. Urban Zones

Rank	Area	Density (inhabitants per hectare)						
	km²	40	50	60	70	80	90	100
		population (000)						
I	1	4	5	6	7	8	9	10
II	3	12	15	18	21	24	27	30
III	10	40	50	60	70	80	90	100
IV	30	120	150	180	210	240	270	300
V	100	400	500	600	700	800	900	1,000
VI	300	1,200	1,500	1,800	2,100	2,400	2,700	3,000
VII	1,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000
VIII	3,000	12,000	15,000	18,000	21,000	24,000	27,000	30,000

At following size-classes, the territory of the inner center, wider center and periphery, as well as the suburban belt is only getting bigger. Thus the cities with population of over 1 million have ***an inner center*** with population of up to 100,000 at the central 10km² area, their ***wider center*** spreads on 90 km² with population of up to 900,000. Their ***periphery*** covers further 200 km², with population of up to 2 million, and their ***inner suburban belt***

[6] The NUTS concept is recognizing the need for dividing this adjacent urbanized area into several zones, but is not using the same multiplier as the one applied here. Also, the threshold values differ considerably from the ones acquired here.

spread on 700km², an ***outer suburban ring*** on additional 2,000 km², with population of many more millions.

4) Is the International Table of Cities "periodic" enough?

To my knowledge, there is only one previous article mentioning the periodic table within spatial context, the one by Woldenberg (1979). But even there the periodic table figurate merely in the headline. The article further elaborates an important aspect (the distribution of cities over river flows, within an urban hierarchy) tackled earlier by Berry and Woldenberg (1966). But its main conclusion about the number of cities per river flow (usually 2 or 3) is not of much help in designing a periodic table of cities.

The other paper, occasionally mentioned in literature in this context, the one by Barr and Leven (1972), does mention size-class (or, rather, scale), and densities, but doesn't even mention the periodic table.

Therefore we have to start from scratch. Since we deliberately modeled ITC after PTE it is no wonder that some formal resemblance has been achieved. The number of rows and columns, and therefore the number of cells within the table is rather similar. What's more, the size-classes within ITC can be up to a point regarded as *periods*, containing a predictable set of city sub-centers. Particularly if we consider rows as different zones of the city area.

We could even try to differentiate the columns by their "deficit" in number of city sub centers, and devote each one to particular group of cities (mountainous, island, coastal, plain, desert), but it doesn't seem to establish the exact connection between such deficit and the particular group of cities. Particularly since the "deficit" in the number of city sib-centers is influenced by other factors, such as the scale economy in retail trade.

Apart from formal resemblances, it is more important that the rise in population numbers and area size brings ***a more complex city economy structure***. From the Central Place Theory it is known that the bundle of (both private and public) goods and services is considerably larger in each of the higher ranked central places (in this case in the cities of higher size-class). From ***the Export Base concept*** it follows that share of the non-base activities is constantly growing with the city size.

City traffic problems also increase with the city size, as well as the need for efficient public transportation, and the need for wider streets. Also, it can be easily noticed that *the average building height* is growing roughly ***by a floor per size-class***. These physical differences reflect those differences in the economy structure and its bundle of goods and services. Within the same urban hierarchies these differences are also reflected in different *wages* and *real-estate prices*.[7]

[7] For similar correlation between the city size and the economic parameters see Parr and Jones (1983)

Consequently these differences are reflected in the size of city GDP too, but due to huge differences in the level of growth between countries, there are no prevailing wages and real estate prices for each of the size classes within ITC.

The tentative List of the cities by their GDP at their first 300 km², is given in Annex III.

Yet, all these differences, despite being more or less noticeable, pale in comparison with the differences between chemical elements. What's more, there is no specific, *predetermined size for each city*, to which it is heading. Thus we can understand this ITC more as ***a series of growth stages*** each city has to pass in order to reach the maximum city size, than a collection of all possible city sizes, each of which have *unique features*, distinguishing it from all other city sizes. More important, even *the same city size* (both in terms of population and area) does not translate into exactly the same features, as is the case with the atoms.

Therefore we may speak of PTC only in a metaphorical sense, to stress not so much those formal similarities with the PTE, but more those considerable differences in the city structure stemming from the differences in city size. It would be useful if further research would improve our knowledge of influence city size have on all aspects of its structure. For instance on the impact of scale economy in retail sector and E-commerce on reduction of the number of retail outlets and consequently on the reduction of the number of city sub centers. *Or, for the relation of city size and the optimal Price-to-Income ratio, and so on.*

In the end we may gain enough knowledge to offer some improved Lowry (1966) model of a city *for each city size* (or at least *a size-class*), with specific number of sub-centers in each size class. Ideally from such a set of outcomes, the members of Tiebout's type club might choose average height of buildings for each city zone, coupled with adequate street width, number of public transportation vehicles, number of public squares and parks, and so forth. This would certainly make urban planning easier.

Although such improvements in our knowledge would not make this ITC "periodical enough" to be labeled as PTC without commas, perceiving cities as constellations of sub-centers distributed across different urban zones in a predictable, regular pattern, within some fixed maximum area for the inner city, should make their international comparison much easier.

5) Densities

Clark (1951) gave an extensive overview of urban densities in the past, and concluded that with the rising GDP/pc, and at a more mature level of urbanization, they tend to fall. Also, at any point in time, densities tend to be highest at some point in the city centre, and then tend to fall down to very low levels at the city border. Clark (1978) also stresses that in many western cities there is a "Crater" at the city's CBD where the density is not as high as in the surrounding zone because the commercial premises have outbid the residential ones. Such a feature is not always present at lower stages of urbanization.

The ***density gradient*** differs considerably from city to city. It is rather steep at earlier stages of urban growth, and then gets constantly flatter. But, in some cases (Sao Paulo, Moscow, Houston), the density gradient is almost flat across the entire Inner City area. Exceptionally, it may actually grow, as in the case of Mumbai, where the density is higher in some suburbs than in the city center. Bertaud (2004) is explaining these exceptions by the urban planning obstacles to the normal functioning of the real estate market, which cannot produce the expected outcome along the bid rent curve theory.

The average densities of the extra large cities for each stage of the Urban Marathon, are given in Annex IV, so that we can track the pace at which they are declining.

5.1. Adding densities into the table

To add densities into the table we have to make a slight change regarding the figures for the areas in each rank (size-class). Instead to regard them as the final value of a range, as was the case in the Table 2.0., we may simply consider them as a fixed threshold value for each of the ranks. In that case, by matching the *population numbers* with those *fixed corresponding areas*, from the table 2.0., we automatically determine *the densities* for each column as is shown in the table 3.0.

Table 3.0. Population numbers and area sizes

Rank	Area	population (000)						
	km²	40	50	60	70	80	90	100
I	1	4	5	6	7	8	9	10
II	3	12	15	18	21	24	27	30
III	10	40	50	60	70	80	90	100
IV	30	120	150	180	210	240	270	300
V	100	400	500	600	700	800	900	1,000
VI	300	1,200	1,500	1,800	2,100	2,400	2,700	3,000
VII	1,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000
VIII	3,000	12,000	15,000	18,000	21,000	24,000	27,000	30,000

It turns out that the densities we get in the process - 40, 50, 60, 70, 80, 90, and 100 persons per hectare, are rather moderate densities for many, if not most of the cities (at least in terms of their continuously built-up areas). Yet, it is obvious that the very low and the very high densities are missing.

But if we try to include them in the table too, we would loose one essential feature - the rows in the table would cease to represent *the same city size-class!* Namely, the rule of threes is limiting the scope of densities that can be represented in the same table. Since densities are varying from as low as 10 persons/ha till over 300 persons/ha, not even a multiplier of 10 (as in the table 0.1.) would help.

The only way to by pass this obstacle seems to be to treat the table 3.0. as "***the core table***", and to add "***extensions***" to this table, one for the very low and the other for the very high densities. Painting the core table in one colour, and the extensions in different ones, as is done in the table 3.1. enables us to save the size-classes feature for rows by reading each segment of this "triptych" separately.

Table 3.1. Extended number of columns

Rank	Area	Density (inhabitants per hectare)												
	km²	10	20	30	40	50	60	70	80	90	100	150	200	300
		population (000)												
I	1	1	2	3	4	5	6	7	8	9	10	15	20	30
II	3	3	6	9	12	15	18	21	24	27	30	45	60	90
III	10	10	20	30	40	50	60	70	80	90	100	150	200	300
IV	30	30	60	90	120	150	180	210	240	270	300	450	600	900
V	100	100	200	300	400	500	600	700	800	900	1,000	1,500	2,000	3,000
VI	300	300	600	900	1,200	1,500	1,800	2,100	2,400	2,700	3,000	4,500	6,000	10,000
VII	1,000	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000	15,000	20,000	30,000
VIII	3,000	3,000	6,000	9,000	12,000	15,000	18,000	21,000	24,000	27,000	30,000			
IX	10,000	10,000	20,000	30,000										
X	30,000	30,000												

In return, this table 3.1. illustrates more clearly an obvious effect of those differences in the density. Namely, that ***cities with higher densities*** can reach the maximum city size already at the area of 100 km², while those with smaller densities may reach this figure only when their area grows above the 1,000 km² mark. Or, rather, that cities with very high densities (100-200-300 inhabitants/ha), are "two size-classes ahead" of cities with very low densities (10-20-30 inhabitants/ha).

The mismatch of the cities with extra small, and the cities with extra high densities is even better illustrated in the table 3.2. where the structure of city sub-centers point to another problem. Namely, the cities with larger densities have, *within the same area*, sub-centers of a higher rank than the cities with average density, and two ranks higher than the cities with low densities. Thus, although they are within the same size class in terms of area, in terms of population they are one, or two size-class ahead.

Consequently, ***in terms of the size of their sub-centers*** they are a size-class ahead of cities with medium densities, and two size-classes ahead of cities with small densities.

Table 3.2. Number of sub-centers for the extended number of densities

Rank	Area	Density (inhabitants per hectare)								
	km²	10	20	30	50	70	100	150	200	300
		population (000)								
I	**1**			**3**	**5**	**7**	**10**	**15**	**20**	**30**
										1x30
							1x10	1x10	2x10	3x10
				1x3	1x3	2x3	3x3	5x3	6x3	10x3
II	**3**	**3**	**6**	**10**	**15**	**21**	**30**	**45**	**60**	**90**
							1x30	1x30	2x30	3x30
				1x10	1x10	2x10	3x10	4x10	6x10	9x10
		1x3	2x3	3x3	5x3	7x3	10x3	15x3	20x3	30x3
III	**10**	**10**	**20**	**30**	**50**	**70**	**100**	**150**	**200**	**300**
										1x300
							1x100	1x100	2x100	3x100
				1x30	1x30	2x30	3x30	5x30	6x30	10x30
		1x10	2x10	3x10	5x10	7x10	10x10	15x10	20x10	30x10
		3x3	6x3	10x3	16x3	23x3	33x3	50x3	66x3	100x3
IV	**30**	**30**	**60**	**100**	**150**	**210**	**300**	**450**	**600**	**900**
							1x300	1x300	2x300	3x300
				1x100	1x100	2x100	2x100	3x100	4x100	6x100
		1x30	2x30	3x30	5x30	7x30	10x30	15x30	20x30	30x30
		3x10	6x10	10x10	15x10	21x10	30x10	45x10	60x10	90x10
		8x3	14x3	33x3	50x3	70x3	100x3	150x3	200x3	300x3
V	**100**	**100**	**200**	**300**	**500**	**700**	**1,000**	**1,500**	**2,000**	**3,000**
										1x3000
							1x1000	1x1000	2x1000	3x1000
				1x300	1x300	2x300	2x300	4x300	4x300	6x300
		1x100	2x100	3x100	5x100	7x100	10x100	15x100	20x100	30x100
		3x30	6x30	10x30	16x30	23x30	33x30	50x30	66x30	100x30
		10x10	20x10	30x10	50x10	70x10	100x10	150x10	200x10	300x10
		33x3	66x3	100x3	163x3	233x3	333x3	500x3	666x3	1000x3
VI	**300**	**300**	**600**	**1,000**	**1,500**	**2,100**	**3,000**	**4,500**	**6,000**	**9,000**
							1x3000	1x3000	2x3000	3x3000
				1x1000	1x1000	2x1000	2x1000	3x1000	4x1000	6x1000
		1x300	2x300	3x300	5x300	7x300	10x300	15x300	20x300	30x300
		3x100	6x100	10x100	15x100	21x100	30x100	45x100	60x100	90x100
		10x30	20x30	33x30	50x30	70x30	100x30	150x30	200x30	300x30
		30x10	60x10	100x10	150x10	210x10	300x10	450x10	600x10	900x10
		100x3	200x3	333x3	500x3	700x3	1000x3	1500x3	2000x3	3000x3

This difference means that the inner center of cities with higher densities has a far larger population (market), than those of the less densely populated cities. Those less densely populated cities are reaching such a large market in the inner center only at proportionally larger ***areas*** – in the next size class(es). On the other hand, this also means that the entire set of city sub-centers in the cities with higher density is far more condensated, or rather far more densely distributed within the same area limit (urban zone).

All this is indirectly confirmed by McMillen and Smith (2003), who found that the number of sub centers is higher in the more densely populated cities.

Table 3.3. Density range over enhanced number of areas for top cities

Rank	Area	Density (inhabitants per hectare)												
		10	20	30	40	50	60	70	80	90	100	150	200	300
		population (000)												
	Inner City													
	inner center													
I	1	1	2	3	4	5	6	7	8	9	10	15	20	30
II	3	3	6	9	12	15	18	21	24	27	30	45	60	90
III	10	10	20	30	40	50	60	70	80	90	100	150	200	300
IV	30	30	60	90	120	150	180	210	240	270	300	450	600	900
	wider center													
IV a	50	50	100	150	200	250	300	350	400	450	500	750	1,000	1,500
IV b	75	75	150	225	300	375	450	525	600	675	750	1,125	1,500	2,250
V	100	100	200	300	400	500	600	700	800	900	1,000	1,500	2,000	3,000
	periphery													
Va	200	200	400	600	800	1,000	1,200	1,400	1,600	1,800	2,000	3,000	4,000	6,000
VI	300	300	600	900	1,200	1,500	1,800	2,100	2,400	2,700	3,000	4,500	6,000	10,000
	Greater City													
Via	500	500	1,000	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000	7,500	10,000	
Vib	750	750	1,500	2,250	3,000	3,750	4,500	5,250	6,000	6,750	7,500	11,250		
VII	1,000	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000			
	LUZ													
VIIa	1,500	1,500	3,000	4,500	6,000	7,500	9,000	10,500						
VIIb	2,000	2,000	4,000	6,000	8,000	10,000	12,000	14,000						
VIIc	2,500	2,500	5,000	7,500	10,000	12,500								
VIII	3,000	3,000	6,000	9,000	12,000									
	Urban Agglomeration													
VIIIa	5,000	5,000	10,000	15,000	20,000									
IX	10,000	10,000	20,000											
	Urban Region													
X	20,000	20,000												
X a	30,000	30,000												

Finally, in order to illustrate the change in the range of densities, particularly in successive supra-city zones of "the extra large cities", we can see in table 3.3. extended both the number of rows and the number of columns. White cells denote that none of those most populous cities have such low densities within the first 300 km². ***The "Crater" effect*** is illustrated by the increase in those white cells as we approach the 30-50km² area, which means that the density range gets quite narrow at this point. Actually it would be this narrow for quite a while if not for some of the Asian populous cities whose total city area is rather small.

The density gradient is illustrated by ***the lack of these extra high densities*** beyond the 300 km² area limit either. Densities beyond that limit become progressively smaller and smaller. Thus, for instance at the 3,000 km² limit, there is no urban area with density higher than 40 inhabitants per hectare, as can be seen in Annex V, where the marginal densities for extra large cities are given for each stage of the Urban Marathon.

Therefore, the population figures of more than 10 million for the final VIII class size in the tables 1.0, 2.0, and 3.0, are merely hypothetical, and not confirmed in the practice. This may be, also, interpreted as a kind of indirect proof that 300 km² area limit for the inner city was not set too low.

6) Conclusion

(1) We may conclude that ***the ITC modeled after PTE is designable***. The design of ITC presented here is, of course, not the only possible one. But after discarding others (one of the logical ones being shown here as the table 0.1.), which appeared to be more cumbersome, this one, following "the rule of threes", seems plausible enough to serve as a framework for international comparison among cities.

(2) Borrowing "the rule of threes" from the Central Place Theory set ***the number of rows*** in the table, dividing ***the city size-classes*** both in terms of population and of city areas. For the Inner City (City Proper), within the first 300 km², six such size classes were found, as well as the two additional ones for the suburban rings located within first 3,000 km².

Distinction between *the net urban area* (roughly equal to the Inner City), and *its hinterland* (the gross urban area) had to be made to facilitate the international comparison of cities belonging to the same size-class. Data on the marginal densities for the extra-large cities suggest that it is appropriate to determine some area limits in order to compare supra-city units (at 1,000 km², or 3,000 km²), as well.

(3) ***Perceiving city as a system of sub-centers*** enabled us to introduce the Central Place Theory into the intra-city context (with central places disguised as city sub-centers). It also helped to decide ***the number of columns***, or rather the size sub-classes. Thus the difference among cities within the same size-class (and the same area limit), is reflected in the number of city sub-centers.

(4) Among the arbitrary decisions which had to be made, ***the minimum city size*** was set at the population of 10,000 settled on an area of just 1km². Settlements that are smaller than this limit, in terms of both population and area, but are recognized as cities by their countries, under their own criteria, are labeled as ***"micro-cities"***.

(5) For ***the maximum city size*** (or rather for the continuously built Inner City), the population threshold was, arbitrarily as well, set at 3 million, and the area threshold at 300 km². Cities which, within their first 300 km², have a higher population than this 3 million threshold are labeled as ***"extra large cities".***

This maximum size was set for the continuously built Inner City, to avoid the problems brought by artificially overstretched hinterlands. Thus the area threshold of 300 km² served as a kind of ***a city-meter*** for comparison of those most populous cities, the list of which is presented in the Annex I.

(6) Interpreting area limits as *fixed figures*, and *not the ranges* from previous to the current size-class, automatically added ***the densities*** along with the population number and areas. Each column now represented a different density. But the problem was that when we applied "the rule of threes" all the densities turned out to be of medium range. The very low and very high densities were missing.

Including them would bring a loss of a key feature, since the rows in the table would cease to represent the same size-class. To preserve this essential feature the low and the high densities had to be presented in additional tables, as a kind of extension of the core table. This suggests that the difference in urban densities has more importance than is generally perceived.

(7) Perception of a ***city as a set of sub centers*** distributed over ***different urban zones*** in a predictable manner, makes the similarity with PTE somewhat more convincing. Yet the **o**bvious *differences in comparison with the PTE* are that cities can be regarded as elements only in the context of an urban hierarchy, that they don't have a unique size all of the time, and that cities of the same size do not have *identical features*, as the natural elements do.

(8) Therefore, more than just a formal replica of PTE, ITC should be regarded as a *collection of stages of urban growth* each city has to pass in order to get from the smallest to the largest city size. Getting an additional sub-center is therefore just an immediate stage goal of each city.

(9) Comparing cities ***within their size-class***, should bring more insight into the relations between city GDP, budget, wages and housing prices.

(10) Hopefully, further research may bring improved Lowry model with all urban planning elements tailored to particular city size in several flavours, for Tiebout-Buchanan club members to choose.

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